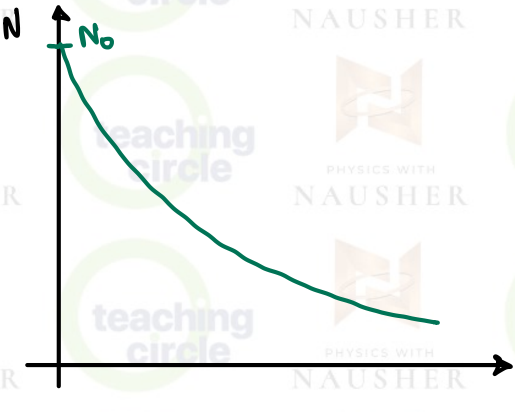
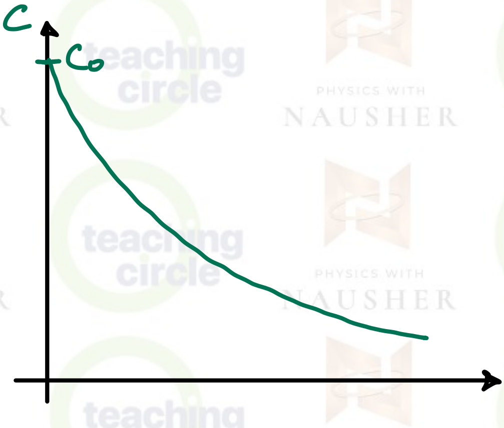
Radioactive Decay equations



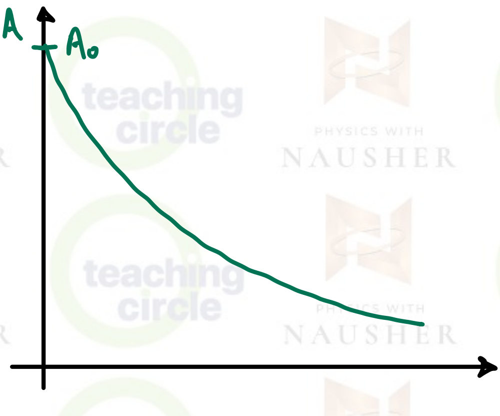
$$N = N_0 e^{-\lambda t}$$

λ: Decay constant
t: time



$$C = C_0 e^{-\lambda t}$$

λ: Decay constant
t: time



$$A = A_0 e^{-\lambda t}$$

λ: Decay constant
t: time

Log Rules

↳ converts non-linear function to linear function

e.g = curve → straight line

1) Product → Sum rule

$c = ab$

$\lg c = \lg(ab) = \lg a + \lg b$

multiplication → addition

2) Power rule

$c = a^n$

$\lg c = \lg a^{n} \xrightarrow[\text{the power}]{\text{bring down}} \lg c = n \lg a$

Examples:-

1) $f = k^2$

$\lg f = 2 \lg k$

2) $f = T^{-1}$

$\lg f = -1 \lg T = -\lg T$

[combine rule ① and rule ②]

- $f = a^n \cdot b^n$

$\lg f = \lg a^n + \lg b^d$

$\lg f = n\lg a + d\lg b$

3. division → subtraction

$h = \frac{f}{a^2} k^e$

- $\lg h = \lg f - \lg a^2 + e\lg k$
- $\lg h = \lg f - 2\lg a + e\lg k$

④ Anti-log

- $c = \lg_{10}(a)$
- $f = \lg a$

$a = 10^f$

Find a

$a = 10^c$

Examples

$2 = \lg a^2$

$2 = 2\lg a \longrightarrow 10^1 = a \rightarrow 10 = a$

One method

$2 = \lg a^3$

$2 = 3^{*}\lg a$

$\frac{2}{3^{*}} = \lg a$

$10^{\frac{2}{3}} = a$

$4.6 = a$

Other method

$2 = \lg a^3$

$10^2 = a^3$

$\sqrt[3]{100} = \sqrt[3]{a^3}$

$4.6 = a$

* To take anti-log, log should be alone.
* if there's something with log, we will move it first.

- base 10 log → $\log_{10}(x)$ → $\lg(x)$

↳ base 10

5) $\lg(10) = 1$

6) Natural log (ln) → $\log_e(x)$ ⇒ $\ln(x)$

↳ base e

(Base e log)

7) $\ln(e) = 1$

① $c = ab$

find $\ln c$

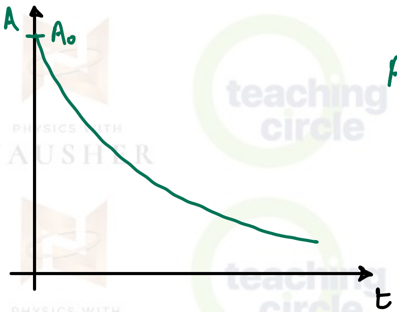
$\ln c = \ln a + \ln b$

② $h = fa^{-2}$

$\ln h = \ln f + (-2\ln a)$

$\ln h = \ln f - 2\ln a$

Linearizing the radioactive graph



$A = A_0 e^{-\lambda t}$

λ: Decay constant
t: time

To convert this exponential graph to linear graph, we need to convert our equation in the form of $y = mx + c$

$A = A_0 e^{-\lambda t}$

To convert this we will multiply ln both sides.

$\ln A = \ln(A_0 e^{-\lambda t})$

$\ln A = \ln A_0 + \ln e^{-\lambda t}$ (Power of e will move down)

$\ln A = \ln A_0 + (-\lambda t)\ln e$ ($\ln e = 1$)

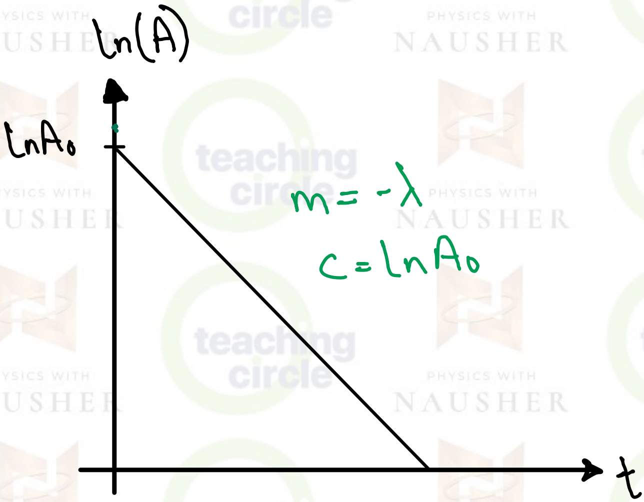
$\ln A = \ln A_0 - \lambda t$

$\ln A = \ln A_0 - \lambda t$

Rearrange this in the form of $y = mx + c$

$\ln A = -\lambda t + \ln A_0$

$y = m x + c$



10 Carbon-15 ($^{15}_{6}C$) is an isotope of carbon that undergoes radioactive decay to nitrogen-15 ($^{15}_{7}N$), which is a stable isotope of nitrogen.

Radioactive decay is both a random and a spontaneous process.

(a) State what is meant by:

(i) random

cannot predict when a nucleus will decay. [1]

(ii) spontaneous

not effected by external factors. [1]

(b) A small sample of carbon-15 decays. The mass *M* of carbon-15 in the sample decreases with time *t*.

Fig. 10.1 shows the variation with *t* of the value of ln (M/ 10^{-16} g).

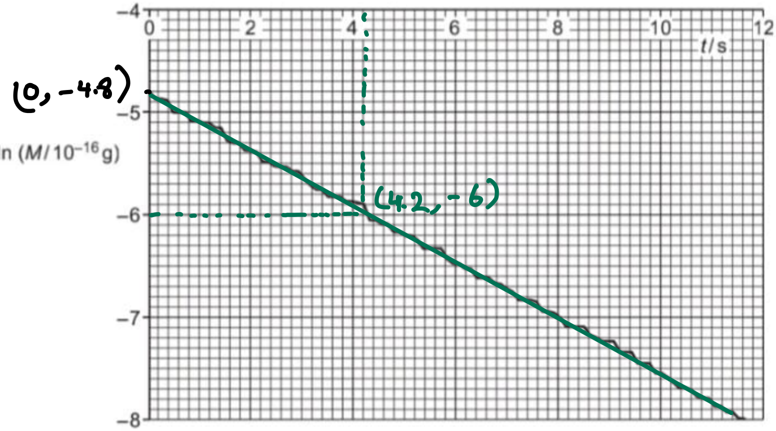


Fig. 10.1

(i) State how Fig. 10.1 demonstrates that radioactive decay is random.

fluctuations in the line. [1]

(ii) On Fig. 10.1, draw the straight line of best fit. [1]

(iii) Show that the decay constant λ of carbon-15 is given by the magnitude of the gradient of your line in **(b)(ii)**.

$M = M_0 e^{-\lambda t}$

$\ln M = \ln M_0 - \lambda t$

$\ln M = -\lambda t + \ln M_0$

$\ln M \to y$, $t \to x$

grad $= -\lambda \Rightarrow \lambda = |\text{gradient}|$

[1]

(iv) Use your line in **(b)(ii)** to determine λ. Give a unit with your answer.

$$\lambda = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-4.8)}{4.2-0} = -0.2857$$

λ = 0.29 unit s^{-1} [2]

(v) Use your answer in **(b)(iv)** to calculate the half-life of carbon-15.

$$\lambda = \frac{\ln 2}{H.L}$$

$$H.L = \frac{\ln 2}{0.29}$$

half-life = 2.43 s [1]

(c) The equation for the decay of carbon-15 can be written as

$$^{15}_{6}C \rightarrow {}^{15}_{7}N + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}.$$

State and explain how the mass of the products of the decay must compare with the mass of the carbon-15 nucleus.

Mass of the decay products will be slightly less than the mass of C-15 because binding energy is released in the reaction.

[2]

[Total: 10]